AD-A229 047

MENTATION PAGE

...is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, ...ng and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this ...ing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson ...s to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

2. REPORT DATE: September, 1990

3. REPORT TYPE AND DATES COVERED: Final Report, 01 Feb 85 to 28 Feb 90

4. TITLE AND SUBTITLE: The Stability and Dynamics of Elastic Structures and Fluid Flows

5. FUNDING NUMBERS: AFOSR-85-0150; 61102F 2304 A4

6. AUTHOR(S): Edward L. Reiss

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES):
Department of Engineering Sciences and Applied Math.
Northwestern University
Evanston, IL 60208

8. PERFORMING ORGANIZATION REPORT NUMBER: AFOSR-TR- 90 1133

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES):
AFOSR/NM
Bldg 410
Bolling AFB DC 20332-6448

10. SPONSORING/MONITORING AGENCY REPORT NUMBER: AFOSR-85-0150

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT: Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

The main thrust of our research program has been the development and applications of asymptotic and perturbation methods for analyzing: the stability and dynamics of elastic structures, fluid flow, and other nonlinear problems; and for problems of scattering of acoustic, electromagnetic and other waves. The work is summarized in the following papers which have been published, accepted for publication, submitted for publication, or are in preparation for publication.

[illegible] Control [illegible] Fluid [illegible]

14. SUBJECT TERMS

15. NUMBER OF PAGES: 23

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAR

Department of Engineering Sciences and Applied Mathematics
Northwestern University
Evanston, IL 60208

Final Report for the Period 2/1/88-9/1/90

Grants: AFOSR 85-0150 and AFOSR-90-0139

The Stability and Dynamics of Elastic Structures and Fluid Flows

Prepared for

Air Force Office of Scientific Research
Bolling AFB, DC 20332

September, 1990

Edward L. Reiss
Principal Investigator

The main thrust of our research program has been the development and applications of asymptotic and perturbation methods for analyzing: the stability and dynamics of elastic structures, fluid flow, and other nonlinear problems; and for problems of scattering of acoustic, electromagnetic and other waves. The work is summarized in the following papers which have been published, accepted for publication, submitted for publication, or are in preparation for publication.

1. Magnan, J. and Reiss, E. L., Rotating Thermal Convection: Neosteady and Neoperiodic Solutions, SIAM J. Appl. Math. 48 (1988), pp. 808-827.

We consider the secondary and cascading bifurcation of a two-dimensional steady and periodic thermal convecton states in a rotating box. We employ previously developed asymptotic and perturbation methods that rely on the coalescence of two, steady convection, primary bifurcation points of the conduction state as the Taylor number approaches a critical value. A multi-time analysis is employed to construct asymptotic expansions of the solutions of the initial-boundary value problem for the Boussinesq theory. The small parameter in the expansion is proportional to the deviation of the Taylor number from its critical value. To leading order, the asymptotic expansion of the solution involves the mode amplitudes of the two interacting steady convection states. The asymptotic analysis yields a first order system of two coupled ordinary differential equations for the slow-time evolution of these amplitudes. We investigate the steady states of these amplitude equations and their linearized stability. These equations suggest that the following sequence of transitions may occur as the Rayleigh number R is increased, for fixed value of the Taylor number near the critical value: First, the conduction state loses stability at a primary bifurcation point to steady convection states (rolls) characterized by a single wavenumber. Then, these states lose stability at a secondary bifurcation point to other steady convection states characterized by two different wavenumbers. Finally, these states become unstable at a tertiary Hopf bifurcation point, $R = R_H$, to time-periodic convection states, which are also characterized by the same two wavenumbers. The Hopf bifurcation is degenerate since for $R = R_H$ there is a one-parameter family of periodic solutions, which is bounded in the phase plane by a heteroclinic orbit. For $R \neq R_H$ but close to R_H, the center of the system's phase plane trajectories is transformed into a focus which is unstable (stable) for $R > R_H$ ($< R_H$). We find that, depending on the length of the observation time and on the initial conditions, the system may appear to be either in a single steady state, or jumping between multiple steady states, or in a periodic state, or in a transient state. If the observation time is sufficiently long the trajectories ultimately spiral into the focus for $R < R_H$ and are thus captured by it; or they spiral out of the unstable focus for $R > R_H$ and thus escape from it. We refer to this type of behavior as neo-periodic capture and escape.

2. Strumolo, G. and Reiss, E. L., Poiseuille Channel Flow with Driven Walls, J. Fluid Mech., submitted.

The effects of a prescribed wall motion on the nonlinear stability of Poiseuille channel flow are studied by an asymptotic method. The motion represents a traveling wave in the upper wall of the channel. It can be considered either as a disturbance to the flow that results from experimental imperfections, or as an externally imposed motion. The

frequency of this disturbance depends on the Reynolds number of the flow. In the classical Poiseuille channel flow problem, the walls are assumed to be rigid. Then a periodic solution bifurcates from the laminar, Poiseuille flow at the critical Reynolds number R_c. In the resonance case, the wall motion destroys the bifurcation. The transition from the laminar state then occurs by jumping at new critical Reynolds numbers. These Reynolds numbers either exceed, or are below R_c, depending on the variation of the wall motion frequency with the Reynolds number. Thus the wall motions can stabilize or destabilize the laminar flow, and hence they can be used to control the transition to turbulence. In the non-resonance cases, the bifurcation is preserved and the critical Reynolds number is slightly perturbed.

3. Kriegsmann, G. A., Bifurcation in Classical Bipolar Transistor Oscillator Circuits, SIAM J. Appl. Math., 49 (1989), pp. 390-403.

The results of a systematic asymptotic analysis of two classical oscillator circuits are presented in this paper. Both circuits are composed of passive components and a single bipolar transistor. They are each described by a third order system of nonlinear, ordinary differential equations which is singularly perturbed in the limit $\epsilon \to 0$. Here ϵ is the inverse of the circuit's "Q". The two-timing method is used to construct the asymptotic behavior of the solutions as $\epsilon \to 0$. These solutions are shown to depend upon amplitudes which evolve, on a slow time scale, according to amplitude equations. These amplitude equations are analyzed and the results are physically interpreted.

4. Erneux, T. and Reiss, E. L., On the Initial Formation of the Oceans and Continents, submitted.

Elastic shell buckling is proposed as a mechanism for the formation of the super continent Pangaea of the Theory of Continental Drift. The primeval lithosphere, whose thickness increased as the earth cooled, is modeled as a thin, elastic, spherical shell, resting on an elastic foundation. This foundation accounts for the softer asthenosphere of the upper mantle. The interior of the earth exerts a centrally directed force on the shell's surface due to its gravitational attraction. It is resisted by the bending and stretching rigidities of the shell and the restoring force of the foundation. It is shown that for plausible values of the physical and geometric properties of the earth, there exists a range of critical values of the thickness of the shell at which buckling could occur. These values are comparable to current estimates of the lithosphere's thickness. Typically, spherical shells snap-buckle into an equilibrium state in which there is a large and localized inward dimple in one hemisphere and the opposite hemisphere remains nearly spherical. The dimple is identified as an ocean and the opposite hemisphere as containing the land mass. The large deformations and stresses that result from snap-buckling may cause the lithospheric material to crack and flow thus suggesting a new driving force for continental drift.

5. Sinay, L. R. and Reiss, E. L., Secondary Transition in Panel Flutter: A Simple Model, in preparation.

A simple, two degree of freedom mechanical model of panel flutter is presented. A perturbation method is employed to determine the secondary bifurcation of flutter states from divergence states, and divergence states from flutter states. The latter suggests a new method for controlling flutter by allowing small amplitude flutter and then increasing the flow velocity until secondary bifurcation into a divergence state occurs.

6. Baer, S. M., Erneux, T. and Rinzel, J., The Slow Passage Through a Hopf Bifurcation. Delay, Memory Effects and Resonance, SIAM J. Appl. Math., 49 (1989), pp. 55-71.

In mathematical studies of bifurcation. it is customary to assume that the bifurcation or control parameter is independent of time. However, in many experiments that are modeled mathematically as bifurcation problems, the bifurcation parameter varies naturally with time, or it is deliberately varied by the experimenter. Frequently, it varies slowly with time or it is forced to vary slowly with time. In this paper, we concentrate on the slow passage through a Hopf bifurcation. As we shall demonstrate, the case of a Hopf bifurcation is quite different from a steady bifurcation or a limit point and our results on the Hopf bifurcation are raising a series of new questions on the control of bifurcation instabilities. We shall consider a specific model for the Hopf bifurcation because our goal is to explore the effects of a slowly varying parameter both analytically and numerically. The model involves several parameters and particular (singular) limits are investigated. These limits reveal interesting features on the slow passage through the bifurcation point.

7. Erneux, T., Q-Switching Bifurcation in a Laser with a Saturable Absorber, J. Optical Soc. of America, B5 (1988), pp. 1063-1068.

In the laser with a saturable absorber (LSA), two spatially separated cells are placed in the laser cavity. The role of the two systems is quite different. One of them is pumped so that the atoms have a positive population inversion (active or amplifying medium); the other one is left with a negative population inversion (passive or absorbing medium). Since the two materials are separated, they will interact only via the field in the cavity. The passive Q-switching operation (PQS) in a LSA consists of repetitive, high intensity, spikes in the output power. Saturable absorbers have been inserted deliberately in many single-mode lasers to produce these short and intense pulses. A number of investigations on specific models have shown that these periodic regimes do not arise from a Hopf bifurcation mechanism. However, the observation of larger periods near a steady bifurcation point strongly suggests that the period solutions emerge from a homoclinic solution (also called infinite period solution or saddle-node separatrix). Periodic solutions emerging from homoclinic solutions have been found near double zero

eigenvalues of the linearized theory and have been analyzed for a large variety of problems. However, they are described by small amplitude elliptic functions which are quite different from the short and intense pulses observed experimentally or numerically. In this paper, we propose a different asymptotic approximation of the periodic solutions which is based on singular perturbation techniques.

8. Erneux, T. and Reiss, E. L., Delaying the Transition of Hopf Bifurcation by Slowly Varying the Bifurcation Parameter, in "Spatial Inhomogeneities and Transient Behavior in Chem. Kinetics," Manchester U. Press (1989), in press.

We explore analytically and numerically the delay or memory effect that occurs when a bifurcation parameter passes slowly through a Hopf bifurcation point and the system's response changes from a slowly varying steady state to slowly varying oscillations. We show for the Brusselator model of spatially homogeneous chemical reactions that the transition is considerably delayed because of the slow variation of the parameter.

9. Mandel, P., Nardone, P. and Erneux, T., Periodic Loss Modulation in a Ring Laser: Influence of Inhomogeneous Broadening and Detuning, J. Optical Soc. of America B5, (1988), pp. 1113-1120.

In a recent paper the influence of periodic loss modulation was studied analytically for a tuned homogeneously broadened ring cavity with unidirectional beam propagation. In this paper we extend these results to assess the influence of inhomogeneous broadening and detuning. Our conclusion is that these two factors do not affect qualitatively the bifurcation diagrams. In addition we determine the bifurcation diagrams when both the pump parameter and the modulation amplitude are taken as control parameters.

10. Cohen, D. S. and Erneux, T., Free Boundary Problems in Controlled Release Pharmaceuticals 1. Diffusion in Glassy Polymers, SIAM J. Appl. Math., 49 (1988), pp. 1466-1474.

We formulate and study two different problems occurring in the formation and use of pharmaceuticals via controlled release methods. These problems involve a glassy polymer and a penetrant, and the central problem is to predict and control the diffusive behavior of the penetrant through the polymer. The mathematical theory yields free boundary problems which we study in various asymptotic regimes.

11. Cohen, D. S. and Erneux, T., Free Boundary Problems in Controlled Release Pharmaceuticals 2. Swelling Controlled Release, SIAM J. Appl. Math., 48 (1988), pp. 1466-1474.

We formulate and study a problem in controlled release pharmaceutical systems. The device we model is a polymer matrix containing an initially immobilized drug. The release of the drug is achieved by countercurrent diffusion through a penetrant solvent with the release rate being determined by the rate of diffusion of the solvent in the polymer. Our mathematical theory yields a free boundary problem which we study in various asymptotic regimes.

12. Erneux, T., Grotberg, J. B., Louhi, B. and Reiss, E. L., Nonlinear Stability Control by External Forcing, in preparation.

Many nonlinear stability problems are formulated mathematically as bifurcation problems. In a typical bifurcation problem there is: a distinguished parameter λ, that is called the bifurcation parameter; and a distinguished solution, which exists for all values of λ, that is called the basic solution. In addition, the bifurcation problem may depend on auxilliary parameters $p = (p_1, p_2, \ldots, p_n)$. The stability of the basic solution is determined from the linearized bifurcation theory. The eigenvalues $\lambda_1(p), \lambda_2(p), \ldots$ are the bifurcation points of the basic solution and they depend, in general, on the auxilliary parameters. The critical value, λ_c, is defined as the smallest bifurcation point of the basic solution. For typical bifurcation problems the basic solution is stable (unstable) if $\lambda < \lambda_c (\geq \lambda_c)$. In the classical methods of stability control, the parameters p are selected so that λ_c is as large as possible. Then the system is used for values of $\lambda < \lambda_c$. Thus, classical stability control employs the linearized bifurcation theory. However, for many problems the condition that $\lambda < \lambda_c$ is too restrictive. In nonlinear stability control (NSC), the solution is permitted to grow and then various techniques are employed to control the response's amplitude so that it is within acceptable limits. Thus a nonlinear theory must be considered. A variety of theoretical and experimental techniques have been proposed to achieve NSC. In this paper we discuss NSC of Hopf bifurcation of periodic solutions by small amplitude, external, periodic forcing functions. A simple model, nonlinear oscillator problem, which is related to the forced van der Pol and Duffing equations, is considered. It has been used as a model of fluid-elastic instability such as occurs in panel flutter, and in elastic tubes conveying fluids. When the amplitude, δ, of the forcing function vanishes then the free oscillation problem has a branch of periodic solutions that bifurcate supercritically from the Hopf bifurcation point at $\lambda = 0$. We study the response of the forced oscillator by considering the forcing function as causing a perturbation of bifurcation, i.e. as an imperfection. Then the method of matched asymptotic expansions, is employed to construct asymptotic expansions of the solutions of the forced oscillator problem

as $\delta \to 0$. The inner expansions of this analysis are obtained by using the multi-time method. We analyze the results to determine if the frequency of the forcing function can be prescribed to delay the transition to Hopf bifurcation at $\lambda = 0$ and to determine the mechanisms of this delay.

13. Erneux, T. and Mandel, P., The Slow Passage Through the Laser First Threshold, Phys. Rev. A, 39 (1989), pp. 5179-5188.

We study the effect of slowly varying cavity losses on the bifurcation diagram of the laser rate equations. Specifically, we consider class B lasers which includes ruby, YAG, CO_2 and semiconductor lasers. We show that the behavior of these lasers depends on two small parameters: v, the .e of change of the cavity decay, and γ, the loss rate for the population inversion. If $v \ll \gamma$, the laser problem reduces to the study of a single nonautonomous equation previously studied by the authors (Phys. Rev. Let. 53, 1818 (1984)). However, if $v = O(\gamma)$ or larger, the response of the laser is described by two strongly coupled nonautonomous equations. We solve these equations in the limit $v = O(\gamma) \to 0$ by using asymptotic methods. We show that (i) the laser admits a new, slowly varying, reference solution which is not a small perturbation of the nonzero intensity steady state solution when the cavity losses are constant and (ii) large amplitude slowly varying oscillations may develop. Our analytical approximations are compared to the exact numerical solution of the laser equations. Finally, we analyze numerically the effects of different initial conditions.

14. Mandel, P., Ruo-ding, L. and Erneux, T., Pulsating Self-Coupled Lasers, Phys. Rev. A., 39 (1989), pp. 2502-2508.

We consider a system of two lasers. The output of each laser is injected in the other laser after a possible attenuation of the field intensity. We formulate the laser equations in the good cavity limit and show that there is a coupling between intensities and phases. If the coupling small, the long0-time response of each laser is time-periodic and is similar to the response of a laser with period loss modulations. Resonance effects are possible and may lead to interesting bifurcation diagrams when the free lasers are different. We investigate the weak coupling limit for arbitrary values of the pump parameters. The resonance problem is analyzed for values of the pump parameters. The resonance problem is analyzed for values of the pump parameters close to the laser first threshold. If the coupling is moderate, stable steady state solutions are possible. The transition from time-periodic to steady state responses appears at a critical coupling amplitude. We propose an asymptotic analysis of the laser equations which is valid for values of the pump parameters close to the laser first-threshold.

15. Erneux, T. and Laplante, J. P., Jump Transition Due to a Time-Dependent Bifurcation Parameter in the Bistable Iodate-Arsenous Acid Reaction, J. Chem. Phys., 90 (1989), pp. 6129-6134.

Bifurcation or limit point problems with slowly varying parameters are ot interest in many research areas. Typically, the time dependency of the bifurcation parameter introduces a delay in the transition from one type of solution to the other. In this article, we study both theoretically and experimentally the bistable iodate-arsenous acid reaction when subjected to a time-dependent bifurcation parameter. This problem is tractable analytically and allows us to derive an expression for the transition delay with respect to the static bifurcation point as a function of the dimensionless ramping rate ϵ. For small and moderate values of ϵ, we predict that the delay is proportional to ϵ^{p} where the exponent $p = 2/3 \approx 0.67$. For the experimental conditions considered in this paper, our experimental data yield $p = 0.63 \pm 0.10$.

16. Mandel, P., Zeghlache, H. and Erneux T., Time Dependent Phase Transition, in "Far From Equilibrium Phase Transitions", L. Garrido, ed. Springer, N.Y., Lecture Notes in Physics No. 319 (1988), pp. 217-235.

We analyze the slow passage through a steady bifurcation point in the presence of small steady, time-periodic and stochastic imperfections.

17. Corron, N. and Kriegsmann, G. A., Saturation Effects in Nonlinear Transition Oscillators, in preparation.

The large signal characteristics of a transistor oscillator are determined bo a large extent by device saturation. The mathematical description of the oscillator is given by a third order system of nonlinear differential equations and the transistor saturation is modelled by an exponential term. The latter is exponentially small if the circuits gain is held small enough. However, when the gain is increased past a point, this term dominates the oscillator's response. To obtain a preliminary idea of this limiting process a model problem is analyzed. It is a van der Pol oscillator with an exponential term that mimics the behavior described above. This problem has been successfully analyzed using singular perturbation techniques and the results have been extended to cover the same effect in real transition oscillators.

18. Kriegsmann, G. A., Mathematical Analysis of a Model Switched-Mode Power Supply, in preparation.

The results of a systematic analysis of a model switched-mode power supply are presented. The physical circuit is modelled by a system of nonlinear differential equations which is singularly perturbed in the limit as $Tf/Ts \to \infty$. Here Ts is the period of the driving switch and Tf is a typical time constant of the associate filter. The system is

nonautonomous in a novel manner. Certain elements in the equations depend upon the driving switch, which is modelled as a periodic binary function, whose state changes at a point determined by one or more of the dependent variables. Asymptotic results are given for the output voltage in the limit described above and questions of stability are addressed.

19. Baker, W. P., Kriegsmann, G. A. and Reiss, E. L., Pulse Scattering by Baffled Cavity-Backed Membranes, J. Acoust. Soc. Amer. 83 (1988), pp. 433-440.

A flexible membrane backed by a rigid cavity is set into an infinite plane baffle. The membrane lies in the plane, and the cavity is in the lower half-space. The upper half-space contains a homogeneous acoustic fluid. The cavity is filled with another homogeneous acoustic fluid. A pulse, which satisfies the acoustic wave equation in the upper half-space, is incident on this baffled membrane. Asymptotic expansions as $\epsilon \to 0$ that are uniformly valid in t are obtained for the membrane's motion, and the scattered and cavity acoustic fields. Here, $\epsilon << 1$ is the ratio of the density of the fluid in the upper half-space to the membrane's density. If the bandwidth of the pulse's spectrum is sufficiently narrow, so that it is free of any of the resonant (natural) frequencies of the membrane or of the cavity, then the pulse is essentially reflected as though the baffled, cavity-backed membrane is rigid. However, if the pulse's spectrum is sufficiently broad, so that it contains cavity and membrane resonant frequencies, an additional scattered field is produced by its interaction with the membrane's motion and the acoustic field inside the cavity. This scattered field insonifies distant observation points after the rigidly reflected pulse arrives. It is a complicated sum of slightly damped and oscillating outgoing spherical waves. The damping occurs on the two time scales ϵt and $\epsilon^2 t$. Some of these damped spherical waves are perturbations, due to the cavity fluid, of the "decayed ringing" of the vacuum-backed, baffled membrane. Application is given to the circular membrane that is backed by a cylindrical cavity, of the same radius as the membrane, and insonified by the normally incident, plane, spiked pulse. Graphs of the membranes motion are presented.

20. Baker, W. P., Kriegsmann, G. A. and Reiss, E. L., Acoustic Scattering by Baffled Cavity-Backed Membranes, J. Acoust. Soc. Amer. 83 (1988), pp. 423-432.

A flexible membrane backed by a rigid cavity is set into an infinite plane baffle. The upper half-space contains an acoustic fluid, and the cavity which lies in the lower half-space, contains another acoustic fluid. A time harmonic wave in the upper half-space is incident on the plane. When the frequency of the incident wave is bounded away from the resonant (natural) frequencies of the membrane, and of the acoustic fluid in the cavity, the reaction of the fluid on the cavity-backed membrane (CBM) is small. However, near a resonant frequency, the fluid-CBM coupling is significant. The method of matched asymptotic expansions is employed to obtain an asymptotic expansion of the scattered field as

$\epsilon \to 0$ that is uniformly valid in the frequency of the incident wave. Here, $\epsilon \ll 1$ is the ratio of the upper-half space fluid and membrane densities. Simple and double resonant frequencies are analyzed; but the method is applicable to higher order resonant frequencies. The method is applied to the normal incidence of a plane wave on a baffled circular membrane that is backed by a circular cylindrical cavity. We find that the CBM is a more "susceptible" scatterer than the vacuum-backed membrane (VBM). In addition, the signature of the CBM differs from the signature of the VBM. For example, the scattered amplitude for the CBM has peaks with bandwidths of $O(\epsilon^2)$, in addition to the peaks with bandwidths of $O(\epsilon)$ corresponding to the membrane resonant frequencies. Moreover, the CBM can have resonant "packets" which are of total bandwidth $\sqrt{\epsilon}$.

21. Kriegsmann, G. A., Norris, A. N. and Reiss, E. L., <u>Pulse Scattering by Penetrable Acoustic Targets</u>, in preparation.

22. Abrahams, I. D., Kriegsmann, G. A. and Reiss, E. L., <u>On the Development of Caustics in Linear Shear Flows Over Rigid Walls</u>, SIAM J. Appl. Math. <u>49</u> (1989), pp. 1652-1664.

    In this paper we examine the scattered field produced by a source inside a linear shear layer flowing above an infinite, rigid wall. The shear layer is topped by a uniform flow. A representation of the solution is obtained in terms of a pair of functions which satisfy a homogeneous second order ordinary differential equation, with variable coefficients. These functions have been studied previously by Jones, and their asymptotic form in the high frequency limit is taken. The representation yields, in this limit, a pair of parametric equations whose solution describe the formation of an infinite set of acoustics inside the shear layer and downstream of the source. This is in agreement with previous results found by employing ray theory techniques. The advantages of the present method and its application to other physical problems are discussed.

23. Abrahams, I. D., Kriegsm[illegible]n, G. A. and Reiss, E. L., <u>On the Development and Control of Caustic[illegible] [illegible] Linear Shear Flows Over Elastic Walls</u>, in preparation.

    The boundary layer ove[illegible] [illegible] infinite elastic surface is modelled by a linear shear layer belo[illegible] [illegible] uniform flow. The fluid is assumed to be inviscid and compressi[illegible] [illegible]d the acoustic field is determined by transform methods whe[illegible] [illegible] [illegible]rce is placed inside the layer. An asymptotic form of th[illegible] [illegible]on is obtained in the high frequency limit and, as for the rigid p[illegible] [illegible]roblem, caustics are found downstream of the source. A full asympto[illegible] analysis is performed to find the sound fields both on and off the ca[illegible]s and in the near and far fields. The elastic plate does not alter th[illegible] [illegible]rder of magnitude estimates found for the rigid plate if the internal damping is ignored. However, if the mechanical damping is included then, fo[illegible] particular values of the plate/fluid

parameters, large attenuation can occur. Results are given for an aluminium plate in the air and the possible relevance of noise suppression to the onset of turbulence is discussed.

24. Abrahams, I. D., Kriegsmann, G. A. and Reiss, E. L., High Frequency Sound Radiation from a Point Source in a Wall Shear Layer, in preparation.

25. Abrahams, I. D., Acoustic Scattering by a Finite Nonlinear Elastic Plate, Part I: Primary, Secondary and Combination Resonances, Proc. Roy. Soc. London A414 (1988), pp. 237-253.

A thin elastic plate of finite width is irradiated by time-harmonic acoustic waves. The fluid is assumed light compared with the plate mass, and the forcing term is of sufficient amplitude to necessitate the inclusion of a nonlinear term (due to mid-plane stretching) in the plate equation. The order one scattered field is determined by the method of multiple scales when the forcing frequency approaches a free oscillation frequency (eigenfrequency) of the plate. This solution is shown to agree with previous work, for the linear problem, and can be multivalued for particular values of the plate-fluid parameters. The scattered wave may also exhibit jumps in its amplitude and phase angle as it varies with frequency, incident wave angle or incident wave amplitude. The nonlinear term further allows the possibility of secondary and combination resonances. These are investigated and the scattered field is shown to contain terms of different frequencies to those of the incident waves. Multivalued solutions and the associated jump phenomenon are again found for these resonant cases.

26. Abrahams, I. D., Acoustic Scattering by a Finite Nonlinear Elastic Plate, Part II: Coupled Primary and Secondary Resonances, Proc. Roy. Soc. London A418, (1988).

A finite thin elastic plate is set in an infinite rigid baffle and the whole is immersed in a compressible inviscid fluid. Plane sound waves are incident on the elastic plate, and the fluid is assumed light compared to the plate density. Nonlinear terms in the plate equation have previously been found to markedly alter the scattered sound field near resonance; and it is shown in this paper that in-plane tension may result in simultaneous primary and secondary resonances. This coincidence of resonances gives rise to two scattered fields, one oscillating at the acoustic forcing frequency and the other at 3 times this frequency. Both terms have amplitudes which are of the same order as the incident wave.

27. Ahluwalia, D. S., Kriegsmann, G. A. and Reiss, E. L., Direct and Inverse Scattering of Acoustic Waves by Low Speed Free Shear Layers, J. Acoust. Soc. of Am., in press.

In this paper, we present a direct method and a simple inverse method which can used to determine the velocity profile of a shear layer. Specifically, we consider an infinite acoustic medium, with constant density and sound speed, containing a free shear layer of finite extent. We probe the free shear layer with a two-dimensional plane wave, incident on the layer from outside, and we assume that the maximum Mach numbers for the flows in the shear layers are small, e.g. $M = 10^{-3}$ to 10^{-4} are typical values for oceanic shear layers. Then, we employ the method of matched asymptotic expansions to obtain an asymptotic expansion of the solution of the direct scattering problem as $M \to 0$ that is uniformly valid in the dimensionless wave number, k, and the angle, α, of the incident plane wave. We find that the reflection coefficient of the scattered wave is proportional to the Fourier transform of the velocity profile of the free shear layer. We invert this transform to obtain an asymptotic approximation of the solution of the inverse scattering problem. That is, given an incident plane wave and measurements of the reflected wave for a sequence of frequencies, we determine approximations of the thickness and flow velocity distribution of the layer. These results may be useful for studies that use acoustic exploration methods to determine oceanographic properties.

28. Abrahams, I.D., Kriegsmann, G. A. and Reiss, E. L., Propagation from a High-Frequency Line Source in a Linear Free Shear Layer, in preparation.

29. Villamizar, V., Kriegsmann, G. A. and Reiss, E. L., Acoustic Scattering from Baffled Membranes that are Backed by Elastic Cavities, Wave Motion, in press.

An elastic membrane backed by a fluid-filled cavity in an elastic body is set into an infinite plane baffle. A time harmonic wave propagating in the fluid in the upper half-space is incident on the plane. It is assumed that the densities of this fluid and the fluid inside the cavity are small compared with the densities of the membrane and of the elastic walls of the cavity, thus defining a small parameter ϵ. Asymptotic expansions of the solution of this scattering problem as $\epsilon \to 0$, that are uniform in the wave number k of the incident wave, are obtained using the method of matched asymptotic expansions. When the frequency of the incident wave is bounded away from the resonant frequencies of the membrane, the cavity fluid, and the elastic body, the resultant wave is a small perturbation (the "outer expansion") of the specularly reflected wave from a completely rigid plane. However, when the incident wave frequency is near a resonant frequency (the "inner expansion") then the scattered wave results from the interaction of the acoustic fluid with membrane, the membrane with the cavity fluid, and finally the cavity fluid with the elastic body, and the resulting scattered field may be "large". The cavity backed membrane (CBM) was previosly analyzed for a

rigid cavity wall. In this paper, we study the effects of the elastic cavity walls on modifying the response of the CBM. For incident frequencies near the membrane resonant frequencies, the elasticity of the cavity gives only a higher order (in ϵ) correction to the scattered field. However, near a cavity fluid resonant frequency, and, of course, near an elastic body resonant frequency the elasticity contributes to the scattered field. The method is applied to the two dimensional problem of an infinite strip membrane backed by an infinitely long rectangular cavity. The cavity is formed by two infinitely long rectangular elastic solids. We speculate on the possible significance of the results with respect to viscoelastic membranes and viscolastic instead of elastic cavity walls for surface sound absorbers.

30. Kath, W. L. and Kriegsmann, G. A., Optical Tunnelling: Radiation Losses in Bent Fiber-Optic Waveguides, IMA J. of Appl. Math. 41, (1988), pp. 85-103.

We present a new method for determining the radiation losses produced when a weakly-guiding optical fiber is bent. In contrast to previous work, we make no assumptions about the type of bending allowed; the center line of the fiber is allowed to follow any path in three dimensional space. In particular, the method clearly shows both the effects due to curvature and torsion. In addition, the result we obtain contains a detailed description of the distribution of the radiated energy.

31. Hobbs, A. K. and Kath, W. L., Losses and Birefrigence for Arbitrarily Bent Optical Fibers, IMA J. of Appl. Math., 44 (1990), pp. 199-219.

We present a calculation to determine the radiation losses for arbitrarily bent weakly-guiding optical fibers. Our results are derived for the full set of vector Maxwell's equations, and show that the rate of energy loss due to radiation differs for the various modes. These results are different from that calculated from a scalar theory, where the so-called linearly polarized (LP) modes are used to approximate the electromagnetic field distributions, showing that in this case scalar theories give incorrect loss rates. We also show that different ranges for the size of the curvature and torsion produce quite different effects. In particular, we show that for smaller amounts of curvature and torsion the modes retain the basic structure of the modes in the straight fiber while losing energy slowly at differing rates, while for larger amounts of curvature and torsion the modes become distorted by the bending.

32. Jiao, J.; Kath, W. L.; Fang, X. and Marhic, M. E., Losses of Infrared Biconcave Metallic Whispering-Gallery Guides, Proc. 4th Int. Conf. on Infrared Phys., ETH Zurich, Switzerland, August, 1988, and J. of Infrared Phys., 29 (1989), pp. 309-321.

The losses for TE and TM waves in infrared biconcave metallic waveguides are calculated. The guiding region is approximated first as a portion of a sphere, and then of an oblate spheroid; Maxwell's equations are then set up in corresponding coordinate systems. They are solved to find the leading-order term of all six field components. These are used to express the boundary conditions at the finite-conductivity wall, and to calculate the fields propagating into the metal. From these, the propagation losses in the guide are calculated. It is found in particular that the TE losses depend only weakly on the curling curvature. Thus it should be possible to make mid-IR waveguides exhibiting losses of a few percent per turn, for most anticipated shape choices.

33. Kath, W. L.; Jiao, J.; Marhic, M. E. and Fang, X., Boundary Layer Analysis of Infrared Whispering Gallery Waveguides, SIAM J. Appl. Math., 50 (1990), pp. 537-566.

The full vector modal solutions and losses of bi-concave infrared waveguides are found, using a boundary layer analysis of Maxwell's equations. The boundary conditions on the surface are given by assuming the metal of the guide has a large (but still finite) conductivity. This method gives the basic modes in terms of products of Airy and parabolic cylinder (Hermite-Gaussian) functions, and also explicitly shows the modal coupling caused by arbitrary bending and twisting of the waveguide. The results also show that while the quasi-TM mode losses are large, the quasi-TE mode losses remain small even with high second curvature. Thus, low-loss flexible infrared waveguides can be fabricated in this manner.

34. Muraki, D. J. and Kath, W. L., Polarization Dynamics for Solitons in Birefringent Optical Fibers, Phys. Lett. A., 139 (1989), 379-383.

Solitons propagating in a nonlinear optical fiber are shown to undergo a rotation of polarization driven by a cross-phase modulation. Analogous to results for dispersionless waves, an amplitude-dependent criterion for the onset of birefringence-induced instabilities is determined.

35. Miksis, M. J. and Ting, L., Scattering of an Incident Wave From an Interface Separating Two Fluids, Wave Motion, 11 (1989), pp. 545-557.

We consider the scattering of an incident pulse from an interface separating two fluids. The interface can be either an elastic membrane or a two-fluid interface with surface tension. By considering the limit where the ratio of acoustic wavelength to the surface wavelength is small, we systematically derived a boundary condition relating the

scattered wave and the surface deformation. This condition is local and can be used to derive a partial differential equation for the deformation of the interface. This equation includes the contribution of the acoustic waves induced by the motion of the interface and once it is solved it can be used to determine the scattered field. At leading order in our analysis we find the plane wave approximation. The addition of the next order terms results in an on surface condition equivalent to that of Kriegsmann and Scandrett. We present numerical calculations to show that our results are in good agreement with the exact numerical solution as well as that of Kriegsmann and Scandrett. Physical situations where the conditions of our analysis are valid are presented.

36. Miksis, M. J. and Ting, L., A Numerical Method for Long Time Solutions of Integro-Differential Systems in Multiphase Flow, Computers and Fluids, 16 (1988), pp. 327-340.

A numerical method to solve a class of integro-differential equations associated with multiphase flow problems is described. The system has at least two time scales, a normal time and a long time. As an initial value problem, the solution is aperiodic in the normal time and approaches a periodic oscillation over a long time. The numerical method is designed to give accurate calculations over the long time. The method is efficient, saving storage space and computational time. Sample numerical calculations and numerical convergence checks are presented to demonstrate the efficiency and accuracy of the method. The system considered describes the forced nonlinear oscillations of a gas bubble in a liquid. We present a systematic numerical study of the phenomenon of the slow growth of the mean bubble radius to a steady value over a long time. The correlation between the phenomena of a sudden burst of large amplitude and resonance in the forced oscillations (at subcritical frequencies) is also studied.

37. Kriegsmann, G. A., Scattering by Acoustically Large Corrugated Planar Surfaces, J. Acoust. Soc. of Am. 88 (1990), pp. 492-495.

The problem of the scattering of acoustic waves by a large, sound soft (hard), corrugated surface in two dimensions is addressed. The surface undulates periodically up to a characteristic length L beyond which it becomes planar. The height of the corrugation is measured by a characteristic length a [illegible] i's period by Λ. The ordering of these scales is taken to be λ [illegible] a << L, where λ is the wavelength of the incident plane acousti[illegible] The method of matched asymptotic expansions is applied [illegible] the problem in the limit as $\epsilon = a/L \to 0$. This approach is both [illegible]ically systematic and physcially intuitive. The results that are ob[illegible]d in the far field are identical to those obtained by using a fi[illegible] beam approximation for a sound hard surface in two dimensions and alm[illegible] the same for a sound soft case; the only difference being a sin[illegible] factor that yields correct boundary behavior. Results for three-dimensional scattering problems are also derived and are compared similarly.

38. Prince, A. S., Grotberg, J. B. and Reiss, E. L., Primary and Secondary Flutter of Leakage Flow Channels, J. Sound and Vib., submitted.

A mathematical model for the nonlinear stability of two dimensional leakage channels is formulated and analyzed. It consists of an infinite channel with flexible elastic walls containing a flowing viscous, incompressible fluid. Two infinite elastic plates are inserted into the channel, parallel to the walls, to form parallel channels. The walls and the parallel plates are modeled by the Von Karman nonlinear plate theory. To simplify the analysis, the fluid viscosity is modeled by Darcy's law rather than the Navier-Stokes law. Plug flow, where the fluid velocity is constant in each channel and the deflection of each wall is constant is a solution of the problem for all values of the flow speed. The stability of the plug flow state is tested by the linearized stability theory. Stability is lost either by divergence or by flutter. The critical value of the flow speed depends on the system parameters and the mode of oscillation, either symmetric or antisymmetric. The nonlinear problem is solved by the Poincare-Linstedt method near the critical flow speed for flutter. One or more branches of flutter solutions bifurcate from this critical speed depending on the multiplicity of the critical speed. It is shown, depending on the ranges of the system parameters, that the bifurcation is either supercritical or subcritical and that the supercritical (subcritical) solutions are stable (unstable). Finally, it is shown using previously developed methods, that secondary bifurcation of flutter states may occur depending on values of the system parameters. In addition, "mode jumping" between flutter states may occur via the secondary bifurcation states, which are then quasi-periodic solutions of the nonlinear problem.

39. Braza, P. A. and Erneux, T., Singular Hopf Bifurcation to Unstable Periodic Solutions in a NMR Laser, Phys. Rev. A. 40 (1989), pp. 2539-2542.

We apply recent developments in the study of singular Hopf bifurcations to describe the complete bifurcation diagram of a simple NMR laser with an injected signal. The branch of periodic solutions appears at a Hopf bifurcation point and may or may not disappear at a homoclinic point. The bifurcation is always subcritical, which suggests that the periodic solutions are all unstable. Our asymptotic analysis is based on the relative values of the fixed parameters in the problem. Our results complement earlier investigations by Holzner et al. [Phys. Rev. A **36**, 1280 (1987)] and by Baugher, Hammack, and Lin [Phys. Rev. A **39**, 1549 (1989)] on the subcritical Hopf bifurcation in a NMR laser.

40. Cohen, D. S., and Erneux T., Changing Time History in Moving Boundary Problems, SIAM J. Appl Math., 50 (1990), pp. 483-489.

A class of diffusion-stress equations modeling transport of solvent in glassy polymers is considered. The problem is formulated as a one-phase

Stefan problem. It is shown that the moving front changes like $\sqrt{t}$ initially but quickly behaves like t as t increases. The t behavior is typical of stress-dominated transport. The quasi-steady state approximation is used to analyze the time history of the moving front. This analysis is motivated by the small time solution.

41. Ruo-Ding, L., Mandel, P. and Erneux, T., Periodic and Quasiperiodic Regimes in Self-Coupled Lasers, Phys. Rev. A. 41, (1990), pp. 5117-5126.

We explore the bifurcation diagrams of self-coupled unidirectional single-mode ring lasers. We focus our attention on nearly identical lasers with an intermediate coupling and in the limit where the atomic polarizations can be adiabatically eliminated. We determine analytically the domains of a stable steady state, bounded by Hopf bifurcations. We study the stability of the emerging branch of periodic solutions and one case determine the presence of a secondary Hopf bifurcation leading to quasiperiodic solutions. These results are complemented by a set of numerically determined bifurcation diagrams that display the behavior of the solutions far from the bifurcation points.

42. Braza, P. A. and Erneux, T., Constant Phase, Phase Drift, and Phase Entrainment in Lasers with an Injected Signal, Phys. Rev. A., 44 (1990), pp. 6470-6479.

In a laser with an injected signal (LIS), the phase of the laser field can be constant (phase locking) or grow unbounded (phase drift) or vary periodically (phase entrainment). We analyze these three responses by studying the bifurcation diagram of the time-periodic solutions in the limit of small values of the population relaxation rate (small γ), small values of the detuning parameters (small Δ and Θ), and small values of the injection field (small y). Our bifurcation analysis has led to the following results: (1) We determine analytically the conditions for a Hopf bifurcation to stable time periodic solutions. This bifurcation appears at $y = y_H$ and is possible only if the detuning parameters are sufficiently large compared to γ [specifically, Δ and Θ must be $O(\gamma^{1/2})$ quantities]. (2) We construct the periodic solutions in the vicinity of of $y = y_H$. We show that the phase of the laser field varies periodically. We then follow numerically this branch of periodic solutions from $y = y_H$ to $y = 0$. Near $y = 0$, the phase becomes unbounded. (3) A transition between bounded and unbounded phase time-periodic solutions appears at $y = y_c$ $(0 < y_c < y_H)$. This transition does not appear at a bifurcation point. We characterize this transition by analyzing the behavior of the phase as $y \to y_c^+$ and as $y \to y_c^-$. (4) We find conditions for a secondary bifurcation from the periodic solutions to quasiperiodic solutions. The secondary branch of solutions is then investigated numerically and is shown to terminate at the limit point of

the steady states. (5) We develop a singular perturbation analysis of the LIS equations valid as $\gamma \to 0$. This analysis allows the determination of a complete branch of bounded time-periodic solutions. We show that the amplitude of these time-periodic solutions becomes unbounded as $y \to 0$ provided that Δ and θ are sufficiently small compared to γ [specifically, Δ and θ are zero or $O(\gamma)$ quantities].

43. Laplante, J. P., Erneux, T. and Georgiou, M. Jump Transiton Due to A Time-Dependent Bifurcation Parameter. An Experimental Numerical and Analytical Study of the Bistable Iodate-Arsenous- Acid Reaction, J. Chem. Phys., in press.

We study the response diagram of the bistable iodate-arsenous acid reaction and investigate analytically and experimentally the jump transitons due to a time-dependent parameter. This parameter is changed linearly in time and its rate of change is characterized by a dimensionless parameter ϵ. We observe that the jump transition does not occur at a limit point of the steady states but is delayed. Asymptotic studies predict that delay is proportional to an $O(\epsilon^{2/3})$ quantity as $\epsilon \to 0$. The $\epsilon^{2/3}$ law is in agreement with the experimental observations of the transition between the lower and upper branches of steady states even if ϵ is not a small quantity. [Erneux and Laplante, J. Chem. Phys. 90, 6129 (1989)]. In this paper, we investigate experimentally the second transition of the iodate-arsenous acid reaction, namely, the transition between the upper and lower branches of the steady states. We have found that the delay does not satisfy the $\epsilon^{2/3}$ law but rather follows an ϵ law. By comparing the experimental results and the results obtained from an analytical and numerical study of the bistable reaction, we show that the values of ϵ used in the experiments are too large to follow the $\epsilon^{2/3}$ law. We conclude that a time dependent control parameter is a new method to investigate the dynamical properties of jump transitions.

44. Erneux, T. and Schwartz, I. B., A New Asymptotic Theory for the Periodically Forced Laser, in "Nonlinear Dynamics in Optical Systems", ed., by N. B. Abrahams, E. M. Garmire and P. Mandel, OSA Proc. , in press.

Sustained relaxation oscillations and irregular spiking have been observed in many periodically modulated lasers. These observations have been sustantiated numerically by recent studies of the laser rate equations. In this paper, we propose a new asymptotic analysis of the laser equations which assumes that the laser oscillations correspond to relaxation oscillations. We identify a large parameter and construct these periodic solutions using singular perturbation techniques. We obtain the equations for the Poincare map and determine the first period doubling bifurcation.

45. Cohen, D. S. and Erneux, T., Moving Boundary Problems in Controlled Release Pharmaceuticals, in "Free boundary Problems: Theory and Application", ed. by J. Chadam and H. Rasmussen, in press.

The simplest method to deliver drugs is to introduce a relatively large quantity of drug into the body by oral or intravenous means. On the average, the drug is administered at the appropriate dosage but, at certain time points, an inappropriate high level of drug can be observed. A more recent method of drug delivery uses various membrane or polymeric devices to deliver drug over an extended period of time a a constant level to the body (from several hours to several years). Of course the drug is delivered at this level whether the body needs it or not. Moreover, it is not yet clear which drugs or bioactive agents are best given by a constant rate. For some drugs, a pattern of input may be optimal. For example, a modulated delivery system controlled by external means may improve the release pattern of insulin.

All these drug delivery problems are formulated mathematically as one-phase moving boundary problems for the concentration of the drug with time-dependent boundary conditions. Since the release rate depends on the behavior of the moving front, the main problem is to predict its time history. This problem is difficult mathematically because the moving front may have a changing time history which precludes the use of a similarity variable.

In this paper, we consider the Higuchi simple model for drug release. The model describes the diffusion of the dissolved drug initially loaded in the polymeric device. We develop an asymptotic method to determine the position of the moving front. We then apply this method to a specific problem involving a time dependent boundary condition.

46. Erneux, T., Reiss, E. L., Holden, L J. and Georgiou, M., Slow Passage Through Bifurcation and Limit Points. Asymptotic Theory and Applications in "Dynamics and Bifurcations", Proceedings of the Conference, Marseille, France, 1990, in press.

This paper summarizes and reviews the literature on asymptotic results for slow passage problems and describe our recent results on Hopf bifurcations.

47. Deller, K., Erneux, T. and Bayliss, A., Asymptotic and Numerical Stability of Brusselator Turbulence, submitted.

We investigate the Brusselator reaction-diffusion equations with periodic boundary conditions. We consider the range of values of the parameters used by Kuramoto in his study of chaotic concentration waves. We determine numerically the bifurcation diagram of the long-time traveling and standing wave solutions using a highly accurate Fourier pseudo-spectral method. For moderate values of the bifurcation parameter, we have found a sequence of instabilities leading either to periodic and quasiperiodic standing waves or, to apparently chaotic regimes.

However, for large values of the control parameter, we have found only uniform time-periodic solutions or time-periodic traveling wave solutions. Our numerical analysis is substantiated by a new asymptotic analysis of the Brusselator equations valid for large values of the control parameter and small diffusion coefficients. We conclude that the chaotic regime is limited to only moderate values of the control parameter.

48. Erneux, T., and Mandel, P., Slow Passaage Through the Laser First Threshold: Influence of the Initial Conditions, in preparation.

We investigate the slow passage through the laser first threshould in the good cavity limit, when the losses are slowly varied in time. We construct an asymptotic solution of the laser intensity equation and analyze the dely of the bifurcation transition. Our analysis is not restricted to the small intensity regime and we show that the dynamics depends critically on the initial value of the control parameter.

49. Erneux T., Edstrom, R. D., and Goldbeter, A., Ultrasensitivity in Biochemical Systems Regulated by Covalent Modification: the Case Where One Converter Enzyme Acts on Two Protein Substrates, in preparation.

Besides allosteric regulation, the covalent modification of proteins represents one of the most important modes of metabolic control. A particularly interesting feature of regulation by covalent modification is to allow for increased sensitivity in response to a given effector. In the basic covalent modification cycle, a protein W is covalently modified into the form W* by enzyme E_1, while the reverse reaction is catalyzed by enzyme E_2. In the common case where covalent modification takes the form of phosphorylation, E_1 and E_2 are a protein kinase and a phosphatase, respectively. The analysis of the amount of protein modified at steady state as a function of the ratio of modification and demodification rates V_1/V_2 showed that the transition curve for the fraction of modified protein can display a sharp threshould when the converter enzymes E_1 and E_2 are saturated by their substrate. Hence the name of zero-order ultrasensitivity given to this phenomenon.

Another feature of covalent modification is that it is often organized in cascades where the product of one modification cycle behaves as the active enzyme that catalyzes the reversible modification of a second substrate in another cycle; the product of this second cycle may in turn catalyze the modification of yet another substrate in a third cycle, etc. Such a cascade organization allows for response accleration and also for sensitivity amplification if appropriate conditions on the kinetics of the successive cycles are met.

In some metabolic cascades, it may occur that one converter enzyme acts on more than one substrate in different modification cycles. Thus, in the glycogenolytic cascade, the cAMP-dependent protein kinase acts on two different substrates, while a smiliar, dual action is observed for a phosphatase. The question arises as to whether the conclusions reached for the case where each converter enzyme acts on one substrate extend to thecase where a common converter enzyme is shared by two substrates in two distince modification cycles. The purpose of this paper is to address this problem and to investigate, in particular, whether zero-order ultrasensitivity also occurs in these conditions.

50. Erneux, T., and Goldbeter, A., Asymptotic Analysis of Threshold Phenomena in Biochemical Systems Regulated by Covalent Modification, in preparation.

Covalent modification systems showing steep transitions in the steady state amount of modified protein have been studied intensively in recent years. The goal is to find how protein activities can rapidly increase or decrease in response to specific signals. The term "ultrasensitivity" is used when the sensitivity of the system to these signals is greater than the standard hyperbolic (Michaelis-Menten) response.

A central difficulty in the mathematical analysis of these systems is to solve coupled nonlinear algebraic equations for the steady state concentrations of the covalently modified proteins. Except for the case of one protein W reversibly modified between W and W*, the solution of the problem cannot be found analytically and must be obtained numerically. The purpose of this paper is to develop an asymptotic method to determine approximative solutions of these nonlinear equations.

51. Baer, S. M., and Erneux, T., Singular Hopf Bifurcation to Relaxation Oscillations II, General Theory, Higher Order Analysis and Low Frequency Oscillations, in preparation.

This paper is a continuation of our study of the singular Hopf bifurcation to relaxation oscillations (SIAM J. Appl. Math. 46, 721-739 (1986)). In our previous analysis, we considered a system of two first order nonlinear equations. In this paper, we consider a larger class of singular Hopf bifurcation problems and show how the bifurcation problem can be reduced to a weakly perturbed conservative system of two first order equations. Second, we develop a new method to analyze the bifurcation diagram of the periodic solutions. The method is particularly useful if the study of several orders of the perturbation analysis is required. We illustrate our method by exploring the bifurcation diagram of the subcritical Hopf bifurcation of the Fitzhugh-Nagumo equations. Finally, we show that other problems exhibiting low frequency relaxation oscillations but which are not bifurcation problems may benefit from our asymptotic analysis of the Hopf bifurcation.

52. Ueda, T. and Kath, W. L., Dynamics of Coupled Solitons in Nonlinear Optical Fibers, Phys. Rev. A. 42 (1990), pp. 563-571.

A systems of copled Nonlinear Schroedinger Equations (NLS) governs the interaction of propagating pulses in two-mode nonlinear optical fibers and directional couplers. Using NLS solitons as trial functions in an averaged Lagrangian formulation, ordinary differential equation approximations for the pulse dynamics are derived. These ODEs give a criterion for two pulses to attract one another and form a bound state; they also describe the dynamics of the complicated oscillations these pulses undergo in this bound state. In addition, the ODE dynamics show that collisions between these pulses are generally inelastic, in that there is an exchange between translational energy and internal energy (due to pulse-width oscillations). The results of the ODE theory are verified by comparison with numerical solutions of the governing partial differential equations.

53. Muraki, D. J. and Kath, W. L., Hamiltonian Dynamics of Solitons in Optical Fibers, in press.

A nonlinear optical fiber whose dispersion relation is weakly-anisotropic in polarization can cause propagating pulses to undergo a self-induced rotation of polarization. The coupling of the cross-polarized optical fields is described by a pair of nonlinear Schrodinger equations and is studied as a Hamiltonian perturbation to an exactly integrable system with soliton solutions. Simplified descriptions involving simple, low-dimensional dynamical systems for near-soliton evolutions are derived which explain the basic rotation of the spatial pulse structure. The results of these dynamical approximations are compared with numerical computations for the full wave equations.

54. Bourland, F. J., Haberman, R. and Kath, W. L., Averaging Methods for the Phase Shift of Arbitrarily Perturbed Strongly Nonlinear Oscillators with an Application to Capture, in press.

Strongly nonlinear oscillators under slowly varying perturbations (not necessarily Hamiltonian) are analyzed by putting the equations into the standard form for the method of averaging. By using the usual near-identity transformations, energy-angle (and equivalent action-angle) equations are derived using the properties of strongly nonlinear oscillators. By introducing a perturbation expansion, a differential equation for the phase shift is derived and shown to agree with earlier results obtained by Bourland and Haberman using the multiple scale perturbation method. The slowly varying phase shift is used (by necessity) to determine the boundary of the basin of attraction for competing stable equilibria, even though these averaged equations are known not to be valid near a separatrix (unperturbed homoclinc orbit).